# Shunt Calibration of Strain Gage Instrumentation

## I. Introduction

The need for calibration arises frequently in the use of strain gage instrumentation. Periodic calibration is required, of course, to assure the accuracy and/or linearity of the instrument itself. More often, calibration is necessary to scale the instrument sensitivity (by adjusting gage factor or gain) in order that the registered output correspond conveniently and accurately to some predetermined input. An example of the latter situation occurs when a strain gage installation is remote from the instrument, with measurable signal attenuation due to leadwire resistance. In this case, calibration is used to adjust the sensitivity of the instrument so that it properly registers the strain signal produced by the gage. Calibration is also used to set the output of any auxiliary indicating or recording device (oscillograph, computer display, etc.) to a convenient scale factor in terms of the applied strain.

There are basically two methods of calibration available — direct and indirect. With direct calibration, a precisely known mechanical input is applied to the sensing elements of the measurement system, and the instrument output is compared to this for verification or adjustment purposes. For example, in the case of transducer instrumentation, an accurately known load (pressure, torque, displacement, etc.) is applied to the transducer, and the instrument sensitivity is adjusted as necessary to register the corresponding output. Direct calibration of instrument systems in this fashion is highly desirable, but is not ordinarily feasible for the typical stress analysis laboratory because of the special equipment and facilities required for its valid implementation. The more practical and widely used approach to either instrument verification or scaling is by indirect calibration; that is, by applying a simulated strain gage output to the input terminals of the instrument. It is assumed throughout this Tech Note that the input to the instrument is always through a Wheatstone bridge circuit as a highly sensitive means of detecting the small resistance changes which characterize strain gages. The behavior of a strain gage can then be simulated by increasing or decreasing the resistance of a bridge arm.

As a rule, strain gage simulation by increasing the resistance of a bridge arm is not very practical because of the small resistance changes involved. Accurate calibration would require inserting a small, ultra-precise resistor in series with the gage. Furthermore, the electrical contacts for inserting the resistor can introduce a significant uncertainty in the resistance change. On the other hand, decreasing the resistance of a bridge arm by shunting with a larger resistor offers a simple, potentially accurate means of simulating the action of a strain gage. This method, known as *shunt calibration*, places no particularly severe tolerance requirements on the shunting resistor, and is relatively insensitive to modest variations in contact resistance. It is also more versatile in application and generally simpler to implement.

Because of its numerous advantages, shunt calibration is the normal procedure for verifying or setting the output of a strain gage instrument relative to a predetermined mechanical input at the sensor. The subject matter of this Tech Note encompasses a variety of commonly occurring bridge circuit arrangements and shunt-calibration procedures. In all cases, it should be noted, the assumptions are made that the excitation for the bridge circuit is provided by a constant-voltage power supply,[1] and that the input impedance of any instrument applied across the output terminals of the bridge circuit is effectively infinite. The latter condition is approximately representative of most modern strain-measurement instruments in which the bridge output is "balanced" by injecting an equal and opposite voltage developed in a separate network. It is also assumed that there are no auxiliary resistors (such as those commonly used in transducers for temperature compensation, span adjustment, etc.) in either the bridge circuit proper or in the circuitry supplying bridge power.

Although simple in concept, shunt calibration is actually much more complex than is generally appreciated. The full potential of this technique for accurate instrument calibration can be realized only by careful consideration of the errors which can occur when the method is misused. Of primary concern are: (1) the choice of the bridge arm to be shunted, along with the placement of the shunt connections in the bridge circuit; (2) calculation of the proper shunt resistance to simulate a prescribed strain level or to produce a prescribed instrument output; and (3) Wheatstone bridge nonlinearity (when calibrating at high strain levels). Because of the foregoing, different shunt-calibration relationships are sometimes required for

[1] In general, the principles employed here are equally applicable to constant-current systems, but the shunt-calibration relationships will differ where nonlinearity considerations are involved.

# Shunt Calibration of Strain Gage Instrumentation

different sets of circumstances. It is particularly important to distinguish between two modes of shunt calibration which are referred to in this Tech Note, somewhat arbitrarily, as *instrument scaling* and *instrument verification*.

In what is described as *instrument scaling*, the reference is to the use of shunt calibration for *simulating the strain gage circuit output* which would occur during an actual test program *when a particular gage in the circuit is subjected to a predetermined strain*. The scaling is normally accomplished by adjusting the gain or gage-factor control of the instrument in use until the indicated strain corresponds to the simulated strain. The procedure is widely used to provide automatic correction for any signal attenuation due to leadwire resistance. In the case of half- and full-bridge circuits, it can also be employed to adjust the instrument scale factor to indicate the surface strain under a singe gage, rather than some multiple thereof. When shunt calibration is used for instrument scaling, as defined here, the procedure is not directly related to verifying the accuracy or linearity of the instrument itself.

By *instrument verification*, in this context, is meant the process of using shunt calibration to *synthesize an input signal* to the instrument which should, for a perfectly accurate and linear instrument, *produce a predetermined output indication*. If the shunt calibration is performed properly, and the output indication deviates from the correct value, then the error is due to the instrument. In such cases, the instrument may require repair or adjustment of internal trimmers, followed by recalibration against a standard such as the our Model 1550A Calibrator. Thus, shunt calibration for instrument verification is concerned only with the instrument itself; not with temporary adjustments in gain or gage factor, made to conveniently account for a particular set of external circuit conditions.

It is always necessary to maintain the distinction between instrument *scaling* and *verification*, both in selecting a calibration resistor and in interpreting the result of shunting. There are also several other factors to be considered in shunt calibration, some of which are especially important in scaling applications. The relationships needed to calculate calibration resistors for commonly occurring cases are given in the remaining sections of the Tech Note as follows:

**Section Content**

II. ***Basic Shunt Calibration***
Derivation of fundamental shunt-calibration equations.

III. ***Instrument Scaling for Small Strains***
Simple quarter-bridge circuit downscale, upscale calibration. Half- and full-bridge circuits.

IV. ***Wheatstone Bridge Nonlinearity***
Basic considerations. Effects on strain measurement and shunt calibration.

V. ***Instrument Scaling for Large Strains***
Quarter-bridge circuit — downscale, upscale calibration. Half- and full-bridge circuits.

VI. ***Instrument Verification***
Small strains. Large strains.

VII. ***Accuracy Considerations***
Maximum error. Probable error.

For a wide range of practical applications, Sections II, III, and VI should provide the necessary information and relationships for routine shunt calibration at modest strain levels. When large strains are involved, however, reference should be made to Sections IV and V. Limitations on the accuracy of shunt calibration are investigated in Section VII. The Appendix to this Tech Note contains a logic diagram illustrating the criteria to be considered in selecting the appropriate shunt-calibration relationship for a particular application.

## II. Basic Shunt Calibration

Illustrated in Figure 1 is the Wheatstone bridge circuit in its simplest form. With the bridge excitation provided by the constant voltage $E$, the output voltage is always equal to the voltage difference between points $A$ and $B$.

$$E_A = E\left(1 - \frac{R_4}{R_4 + R_3}\right)$$

$$E_B = E\left(1 - \frac{R_1}{R_1 + R_2}\right)$$

And,

$$e_O = E_A - E_B = E\left(\frac{R_1}{R_1 + R_2} - \frac{R_4}{R_4 + R_3}\right) \quad (1)$$

Or, in more convenient, nondimensional form:

$$\frac{e_O}{E} = \frac{R_1 / R_2}{R_1 / R_2 + 1} - \frac{R_4 / R_3}{R_4 / R_3 + 1} \quad (1a)$$

It is evident from the form of Equation (1a) that the output depends only on the resistance ratios $R_1/R_2$ and $R_4/R_3$, rather than on the individual resistances. Furthermore, when $R_1/R_2 = R_4/R_3$, the output is zero and the bridge is described as resistively balanced. Whether the bridge is balanced or unbalanced, Equation (1a) permits calculating the change

## Shunt Calibration of Strain Gage Instrumentation

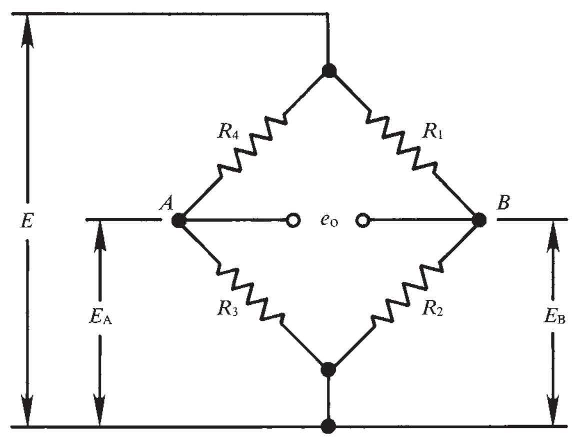



Figure 1 – Basic Wheatstone bridge circuit.

in output voltage due to decreasing any one of the arm resistances by shunting. The equation also demonstrates that the sign of the change depends on which arm is shunted. For example, decreasing $R_1/R_2$ by shunting $R_1$, or increasing $R_4/R_3$ by shunting $R_3$ will cause a negative change in output. Correspondingly, a positive change in output is produced by shunting $R_2$ or $R_4$ (increasing $R_1/R_2$ and decreasing $R_4/R_3$, respectively).

Equation (1a) is perfectly general in application to constant-voltage Wheatstone bridges, regardless of the values of $R_1$, $R_2$, $R_3$ and $R_4$. In conventional strain gage instrumentation, however, at least two of the bridge arms normally have the same (nominal) resistance; and all four arms are often the same. For simplicity in presentation, without a significant sacrifice in generality, the latter case, known as the "equal-arm bridge", is assumed in the

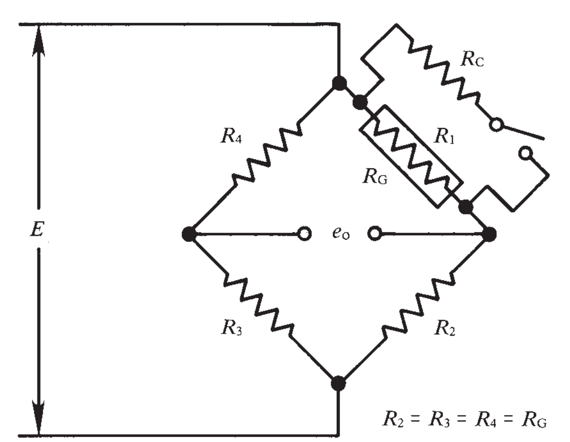



Figure 2 – Shunt calibration of single active gage.

following, and pictured in Figure 2. The diagram shows a single active gage, represented by $R_1$, and an associated calibration resistor, $R_C$, for shunting across the gage to produce an output signal simulating strain. The bridge is assumed to be in an initial state of resistive balance; and all leadwire resistances are assumed negligibly small for this introductory development of shunt-calibration theory. Methods of accounting for leadwire resistance (or eliminating its effects) are given in Section III.

When the calibration resistor is shunted across $R_1$, the resistance of the bridge arm becomes $R_1 R_C/(R_1 + R_C)$, and the change in arm resistance is:

$$\Delta R = \frac{R_1\ R_C}{R_1+R_C} - R_1 \tag{2}$$

Or,

$$\frac{\Delta R}{R_1} = \frac{-\ R_1}{R_1+R_C} \tag{3}$$

Reexpressing the unit resistance change in terms of strain yields a relationship between the simulated strain and the shunt resistance required to produce it. The result is usually written here in the form $R_C = f(\varepsilon_s)$, but the simulated strain for a particular shunt resistance can always be calculated by inverting the relationship.

The unit resistance change in the gage is related to strain through the definition of the gage factor, $F_G$ (see Footnote 2).

$$\frac{\Delta R}{R_G} = F_G\varepsilon \tag{4}$$

where:

$R_G$ = the nominal resistance of the strain gage (e.g., 120 ohms, 350 ohms, etc.).

Combining Eqs. (3) and (4), and replacing $R_1$ by $R_G$, since there is no other resistance in the bridge arm,

$$F_G\varepsilon_s = \frac{-R_G}{R_G+R_C}$$

Or,

$$\varepsilon_S = \frac{-R_G}{F_G\left(R_G + R_C\right)} \tag{5}$$

where: $\varepsilon_s$ = strain (compressive) simulated by shunting $R_G$ with $R_C$. Solving for $R_C$,

[2] In this Tech Note, the symbol $F_G$ represents the gage factor of the strain gage, while $F_I$ denotes the setting of the gage factor control on a strain indicator.

TECH NOTE

# Shunt Calibration of Strain Gage Instrumentation

$$R_C = \frac{R_G}{F_G \varepsilon_S} - R_G \qquad (6)$$

Since the simulated strain in this mode of shunt calibration is always negative, it is common practice in the strain gage field to omit the minus sign in front of the first term in Equation (6), and write it as:

$$R_C = \frac{R_G}{F_G \varepsilon_S} - R_G = \frac{R_G \times 10^6}{F_G \varepsilon_{S(\mu)}} - R_G \qquad (7)$$

where: $\varepsilon_s(\mu)$ = simulated strain, in microstrain units.

When substituting into Equation (7), the user must always remember to substitute the *numerical value* of the compressive strain, without the sign.

The relationships represented by Equations (5) through (7) are quite general, and accurately simulate the behavior of a strain gage for any magnitude of compressive strain. For convenient reference, Table 1 lists the appropriate shunt-calibration resistors for simulating strains up to 10000$\mu\varepsilon$ in 120-, 350-, and 1000-ohm gage circuits, based on a gage factor of 2.000. Precision resistors (±0.02%) in these and other values are available from Vishay Micro-Measurements, and are described in our Strain Gage Accessories Data Book. If the gage factor is other than 2.000, or if a nonstandard calibration resistor is employed, the simulated strain magnitude will vary accordingly. The true magnitude of simulated strain can always be calculated by substituting the exact values of $F_G$ and $R_C$ into Equation (5).

While Equations (5) through (7) provide for accurately simulating strain gage response at any compressive strain level (as long as the gage factor remains constant), this may not be sufficient for some calibration applications. It is always necessary to consider the effects of the Wheatstone bridge circuit through which the instrument receives its input signal from the strain gage. If the nondimensional output voltage of the bridge ($\varepsilon_o / E$) were exactly proportional to the unit resistance change $\Delta R / R_G$, a perfectly accurate instrument should register a strain equal to the simulated strain (at the same gage factor). In fact, however, the Wheatstone bridge circuit is slightly nonlinear when a resistance change occurs in only one of the arms (see Reference 1: Our Tech Note TN-507). Because of this, the instrument will register a strain

Table 1 – Shunt Calibration Resistors

| GAGE CIRCUIT | RESISTANCE IN OHMS | EQUIVALENT MICROSTRAIN[3] |
|---|---|---|
| 120-OHM | 599 880 | 100 |
| | 119 880 | 500 |
| | 59 880 | 1000 |
| | 29 880 | 2000 |
| | 19 880 | 3000 |
| | 14 880 | 4000 |
| | 11 880 | 5000 |
| | 5880 | 10 000 |
| 350-OHM | 349 650 | 500 |
| | 174 650 | 1000 |
| | 87 150 | 2000 |
| | 57 983 | 3000 |
| | 43 400 | 4000 |
| | 34 650 | 5000 |
| | 17 150 | 10 000 |
| 1000-OHM | 999 000 | 500 |
| | 499 000 | 1000 |
| | 249 000 | 2000 |
| | 165 666 | 3000 |
| | 124 000 | 4000 |
| | 99 000 | 5000 |
| | 49 000 | 10 000 |

[3] The "Equivalent Microstrain" column gives the true compressive strain, in a quarter-bridge circuit, simulated by shunting each calibration resistor across an active strain gage arm of the exact indicated resistance. These values are based on a circuit gage factor setting of 2.000.

### Small versus Large Strain

With respect to shunt calibration, at least, the distinction between small and large strains is purely relative. Somewhat like beauty, it resides primarily in the eye of the beholder — or the stress analyst.

Errors due to Wheatstone bridge nonlinearity vary with the circuit arrangement, and with the sign and magnitude of the simulated strain. As shown in TN-507, the percentage error in each case is approximately proportional to the strain. Thus, if the error at a particular strain level is small enough relative to the required test precision that it can be ignored, the strain can be treated as small. If not, the strain is large, and the nonlinearity must be accounted for to calibrate with sufficient accuracy.

Since the nonlinearity error at 2000$\mu\varepsilon$ is normally less than 0.5%, that level has been taken arbitrarily as the upper limit of small strain for the purposes of this Tech Note. The reader should, of course, establish his or her own small/large criterion, depending on the error magnitude compared to the required precision. The accuracy of the shunt calibration precedure itself (see Section VII) should be considered when making such a judgment.

# Shunt Calibration of Strain Gage Instrumentation

which differs from the simulated (or actual) strain by the amount of the nonlinearity error introduced in the bridge circuit. As a rule of thumb, the nonlinearity error in this case, expressed in percent, is about equal to the strain, in percent. Thus, at low strain levels (below, say, 2000$\mu\varepsilon$, or 0.2% — see inset), the difference between the simulated and registered strain magnitudes may not be detectable. For accurate shunt calibration at higher strain levels, or for precise evaluation of instrument linearity, different shunt-calibration relationships may be required. Treatment of nonlinearity considerations is given in Section IV of this Tech Note.

The procedures described up to this point have referred only to instrument calibration for compressive strains. This seems natural enough, since shunting always produces a decrease in the arm resistance, corresponding to compression. There are occasions, however, when upscale (tension) calibration is more convenient or otherwise preferable. The easiest and most accurate way to accomplish this is still by shunt calibration.

Figure 3 illustrates the simple Wheatstone bridge circuit again, but with the calibration resistor positioned to shunt the adjacent bridge arm. $R_2$ (usually referred to as the "dummy" in a quarter-bridge circuit). As demonstrated by Equation (1a), a decrease in the resistance of the adjacent arm will produce a bridge output opposite in sign to that obtained by shunting $R_1$, causing the instrument to register a tensile strain. Thus, a simulated compressive strain ($\varepsilon_{SC2}$) in $R_2$, generated by shunting that arm, can be interpreted as a simulated tensile strain ($\varepsilon_{ST1}$) in $R_1$. The special subscript notation is temporarily introduced here because the two simulated strains are not exactly equal in magnitude. For calibration at low strain levels, the difference in magnitude between $\varepsilon_{ST1}$ and $\varepsilon_{SC2}$ is small enough that the relationships given in Equations (5) through (7) are sufficiently accurate

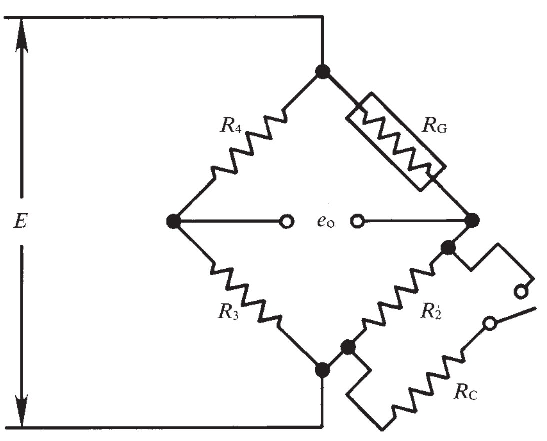


Figure 3 – Upscale (tensile) calibration by shunting adjacent bridge arm.

for most practical applications. The error in the simulated tensile strain, in percent, is approximately equal to the gage factor times the strain, in percent.

The foregoing error arises because shunting $R_2$ to produce a simulated compressive strain in that arm, and then interpreting the instrument output as due to a simulated tensile strain in $R_1$, involves effectively a two-fold simulation which is twice as sensitive to Wheatstone bridge nonlinearity. Accounting for the nonlinearity, as shown in Sections IV and V, permits developing a shunt-calibration relationship for precisely simulating tensile strains of any magnitude.

## III. Instrument Scaling for Small Strains

Very commonly, when making practical strain measurements under typical test conditions, at least one active bridge arm is sufficiently remote from the instrument that the leadwire resistance is no longer negligible. Under these circumstances, the strain gage instrument is "desensitized"; and the registered strain will be lower than the gage strain to an extent depending on the amount of leadwire resistance. In a three-wire quarter-bridge circuit, for instance, the signal will be attenuated by the factor $R_G/(R_G + R_L)$, where $R_L$ is the resistance of one leadwire in series with the gage. The usual way of correcting for leadwire desensitization is by shunt calibration — that is, by simulating a predetermined strain in the gage, and then adjusting the gage factor or gain of the instrument until it registers the same strain.

This section includes a variety of application examples involving quarter-, half-, and full-bridge strain gage circuits. In all cases treated here, it is assumed that strain levels are small enough relative to the user's permissible error limits that Wheatstone bridge nonlinearity can be neglected. Generalized relationships incorporating nonlinearity effects are given in subsequent sections.

### Quarter-Bridge Circuit

Figure 4 illustrates a representative situation in which an active gage, in a three-wire circuit, is remote from the instrument and connected to it by leadwires of resistance $R_L$. If all leadwire resistances are nominally equal, then $R_1 = R_L + R_G$ and $R_2 = R_L + R_G$; i.e., the same amount of leadwire resistance is in series with both the active gage and the dummy. There is also leadwire resistance in the bridge output connection to the S– instrument terminal. The latter resistance has no effect, however, since the input impedance of the instrument applied across the output terminals of the bridge circuit is taken to be infinite. Thus, no current flows through the instrument leads.

To calibrate in compression, the active gage is shunted by a calibration resistor calculated from Equation (7) or selected from Table 1 for the specified strain magnitude.

TECH NOTE

# Shunt Calibration of Strain Gage Instrumentation

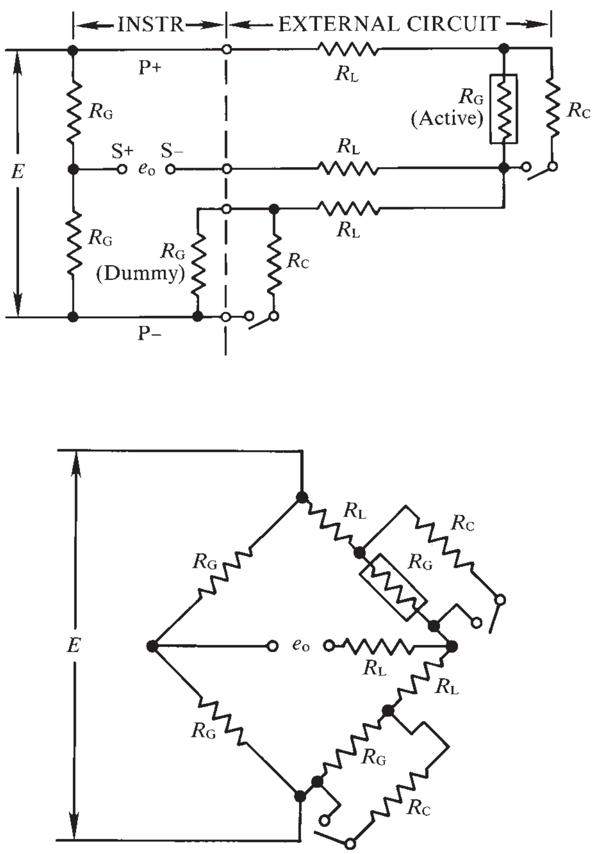



Figure 4 – Quarter-bridge circuit with active gage remote from instrument.

After adjusting the sensitivity of the instrument to register the calibration strain, the effect of the leadwire resistance is eliminated from all subsequent strain measurements.

Unless additional leadwires are used (as demonstrated in Figure 6), simulated compressive strain by directly shunting the remote active gage is usually difficult to implement in practice. Since the purpose of shunt calibration in this case is simply to scale the instrument sensitivity as a means of compensating for leadwire resistance, either upscale or downscale calibration is equally suitable. Thus, it is generally more convenient to shunt the adjacent dummy arm as shown in Figure 4, because this can be done right at the instrument terminals. It should be apparent from the figure that the calibration resistor must be connected directly across the dummy to produce the desired result. Gage strain cannot be accurately simulated by shunting from S– to P– (or from S– to P+). After shunting the dummy with a calibration resistor selected to simulate the appropriate strain, the instrument sensitivity is adjusted to register the same strain. At low strain levels, the result is effectively the same as if the calibration had been performed by shunting the active gage.

## Half-Bridge Circuit

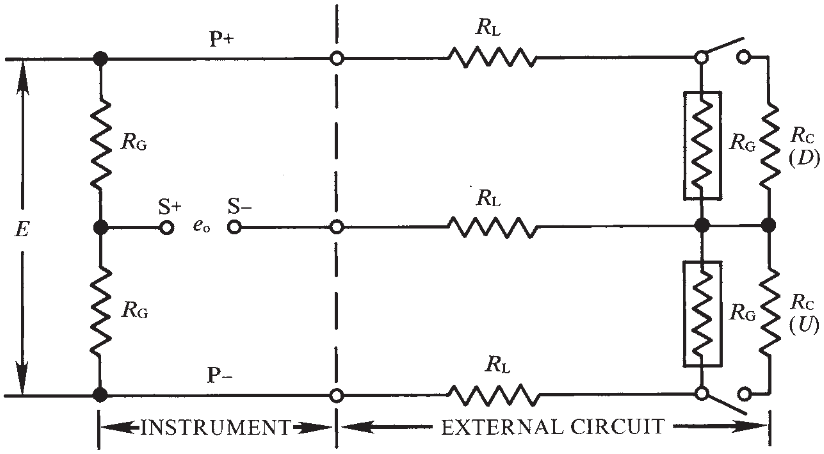
In many stress analysis applications it is necessary (or at least advantageous) to employ two co-acting gages, connected at adjacent arms in the bridge circuit, to produce the required strain signal. A common example of this occurs when a second gage is installed on an unstressed specimen of the test material (and maintained in the same thermal environment as the test object) to provide temperature compensation for the active gage. In the special case of a purely uniaxial stress state, with the principal stress directions known, both gages can be mounted adjacent to each other, directly on the test part. One gage is aligned with the applied stress, and the other is installed in the perpendicular direction to sense the Poisson strain. This arrangement provides an augmented bridge output, along with excellent temperature compensation. Similar opportunities are offered by a beam in bending. One gage is mounted along the longitudinal centerline of the convex surface, with a mating gage at the corresponding point on the concave surface. When the two gages are connected as adjacent arms in the bridge circuit, and assuming uniform temperature through the thickness of the beam, the bridge output is doubled while maintaining temperature compensation.

All of the foregoing are examples of half-bridge circuits, since one-half of the Wheatstone bridge is external to the instrument. Aside from differences in the quality of the achievable temperature compensation, they differ principally in their degrees of signal increase. The factor of signal augmentation is usually expressed in terms of the "number of active gages", $N$. When the gage in the adjacent bridge arm senses no applied strain, but serves solely for temperature compensation, $N = 1$. With two perpendicular gages, aligned along the principal axes in a uniaxial stress field, $N = 1 + \nu$, where $\nu$ is the Poisson's ratio of the test material. In the case of the beam, with gages on opposing surfaces, $N = 2$, since the gages sense equal and opposite strains, and the bridge output is doubled.

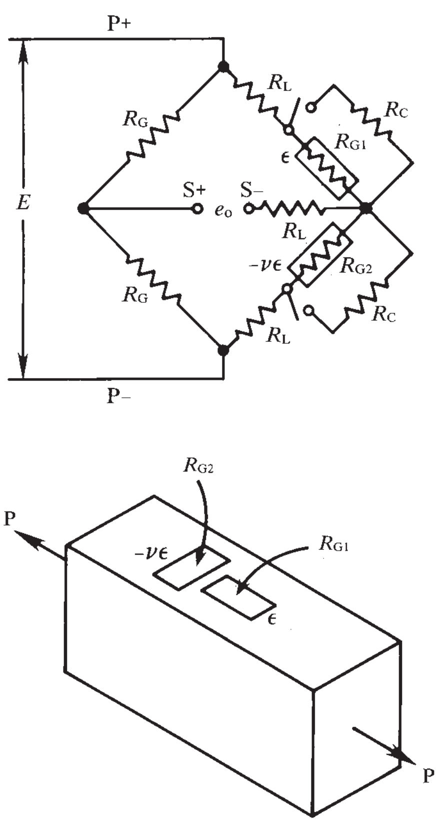
When $N$ is greater than unity, it is obviously necessary to adjust the instrument sensitivity by the factor $1/N$ if the instrument is to directly register the actual surface strain sensed by the primary active gage. Furthermore, if the gage installations are at a distance from the instrument, additional adjustment of the sensitivity (in the opposite direction) is required to compensate for the signal loss due to leadwire resistance. Shunt calibration can correct for both effects simultaneously, and permit adjusting instrument sensitivity to register the correct surface strain at the primary active gage.

# Shunt Calibration of Strain Gage Instrumentation

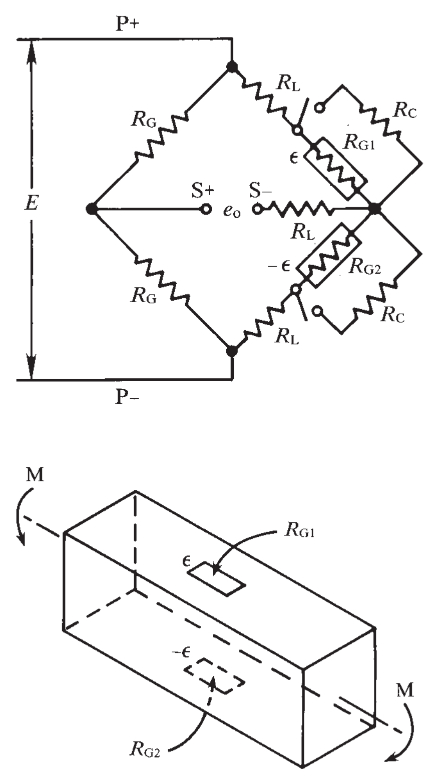
Figure 5 – Shunt calibration of external half-bridge circuits.




5a – Basic circuit.




5b – Uniform uniaxial stress: N = 1 + ν




5c – Bending beam: N = 2

Figure 5a illustrates a typical half-bridge circuit, with the gages located away from the instrument, and with shunt resistors for downscale ($D$) and upscale ($U$) calibration. Figures 5b and 5c show the physical and circuit arrangements for $N = 1 + \nu$ and $N = 2$, respectively. The procedure for calibration is the same as for the quarter-bridge circuit. That is, a calibration resistor of the appropriate size is shunted across the gage, and the instrument sensitivity is adjusted to register the simulated strain.

Confusion sometimes arises, however, in correlating the registered strain with the simulated strain (and with the calibration resistor) when $N$ is greater than unity. The

# Shunt Calibration of Strain Gage Instrumentation

simplest way of handling this is to generalize Equation (7) so that it includes the number of active gages, Thus,

$$R_C = \frac{R_G \text{ x } 10^6}{F_G \text{ x } N \text{ x } \varepsilon_{S(\mu)}} - R_G \qquad (8)$$

To calibrate, the strain gage is shunted with a resistor calculated from Equation (8) and the instrument sensitivity adjusted to register $\varepsilon_s(\mu)$. The result is the same, except for the sign of the instrument output, no matter which of the two adjacent-arm gages in Figure 5 is shunted. After calibration, the instrument output will correspond to the surface strain at the primary active gage. This procedure accounts for both the signal increase (when $N > 1$) and the leadwire desensitization.

When the gage installations are more than a few steps away from the instrument, it is usually inconvenient to connect a shunt-calibration resistor directly across the gage as shown in Figure 5. For such cases, remote shunt calibration is a common practice. Figure 6 illustrates a half-bridge circuit with the calibration resistor positioned at the instrument. In this example, three extra leadwires and a switch permit connecting the shunt across either arm of the half bridge. Since shunt resistors are characteristically in the thousands of ohms, the resistances of the calibration leadwires, although shown in the figure, can usually be neglected in the strain simulation calculations. Equation (8) is then directly applicable to remote shunt calibration. If the leadwire resistance is large enough so that 100 x $R_L/R_C$ is greater than about 1/10 of the required calibration precision (expressed in percent), Equation (8) can be modified as follows to calculate the correct calibration resistance:

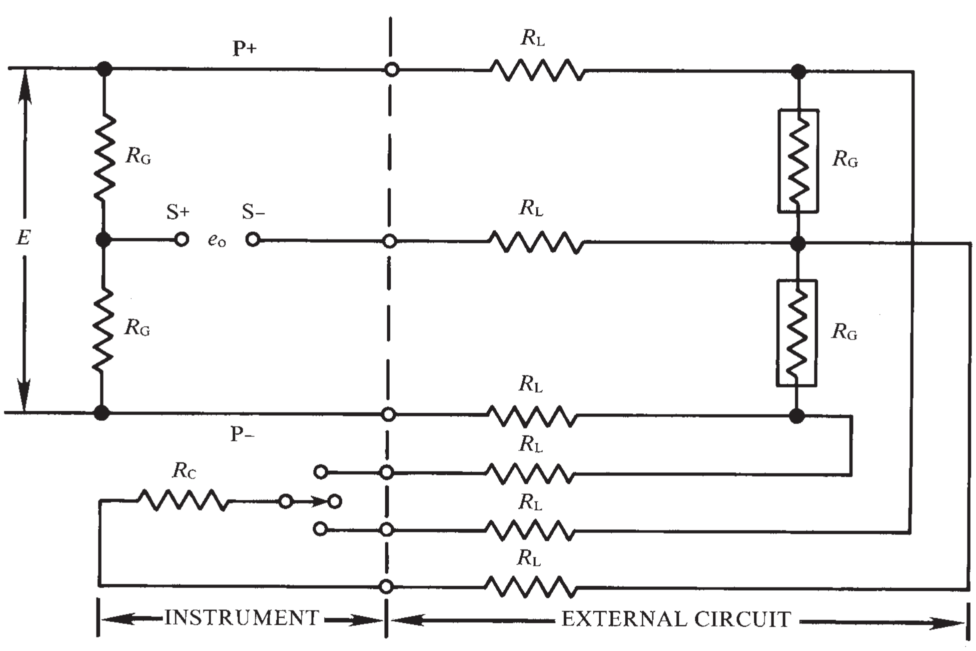
$$R_C = \frac{R_G \text{ x } 10^6}{F_G \text{ x } N \text{ x } \varepsilon_{S(\mu)}} - R_G - 2R_L \qquad (9)$$

In Equation (9), $R_L$ represents the resistance of one leadwire between the calibration resistor and gage.

## Full-Bridge Circuits

In strain-measurement (stress analysis) applications for which the half bridge is suitable, the output signal can be doubled by installing a full bridge, with four active strain gages on the test object. A representative circuit, including two supplementary leadwires for remote shunt calibration, is shown in Figure 7a. In practice, the calibration leadwires can be connected across any arm of the bridge, and will always produce the same signal magnitude, but the sign of the signal depends on which arm is shunted. It will be noticed, in the case of the full-bridge circuit, that the leadwire resistance is now in the bridge power and output leads rather than in the bridge arms. With the assumption of infinite impedance at the bridge output, the resistance in the latter leads has no effect. However, the resistance in the power leads reduces the voltage applied to the bridge proper, and attenuates the output signal accordingly.

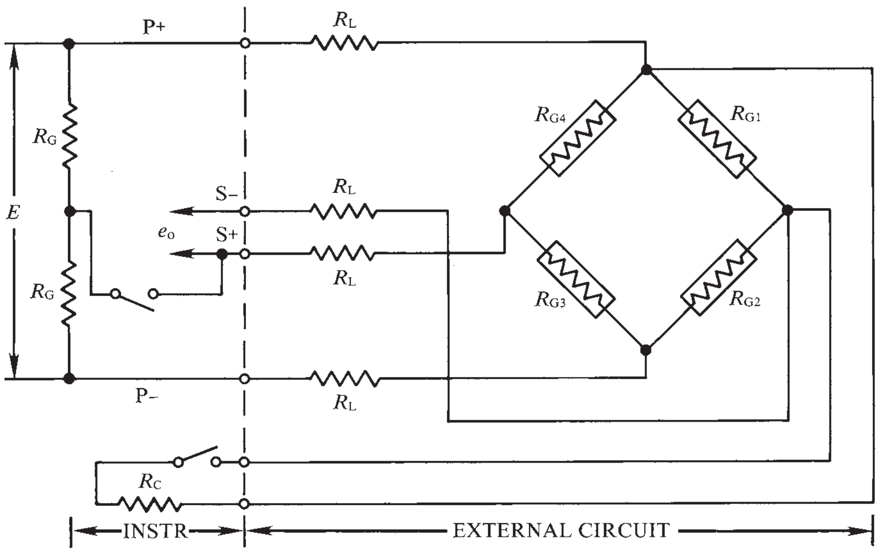
Three widely used full-bridge circuit arrangements are shown in Figures 7b, 7c, and 7d. In the first two of these, for




6 – Remote shunt calibration of external half-bridge.

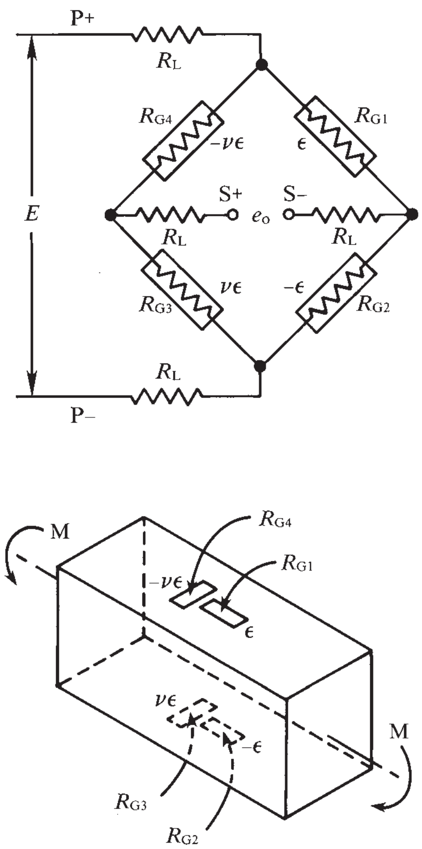

For technical support, contact
micro-measurements@vishay.com
Document Number: 11064
Revision 16-July-07

# Shunt Calibration of Strain Gage Instrumentation

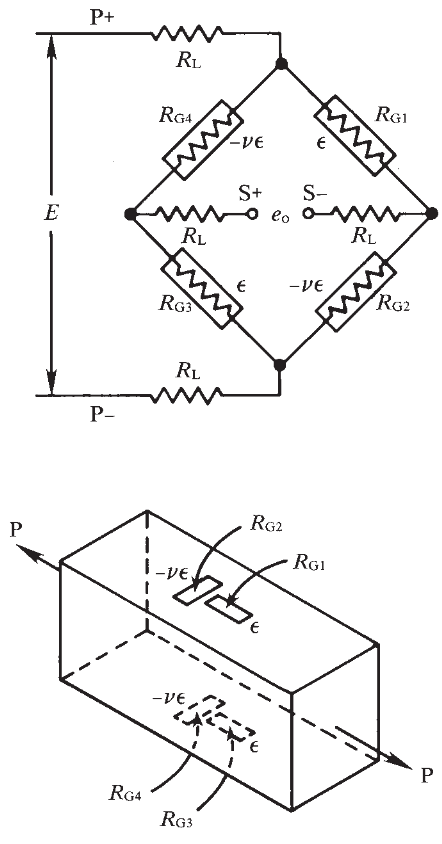



7 – Remote shunt calibration of external full-bridge circuit.

7a – General circuit arrangement. Calibration resistor can be shunted across any arm. Note that internal half-bridge is disconnected from output circuit for full-bridge operation.



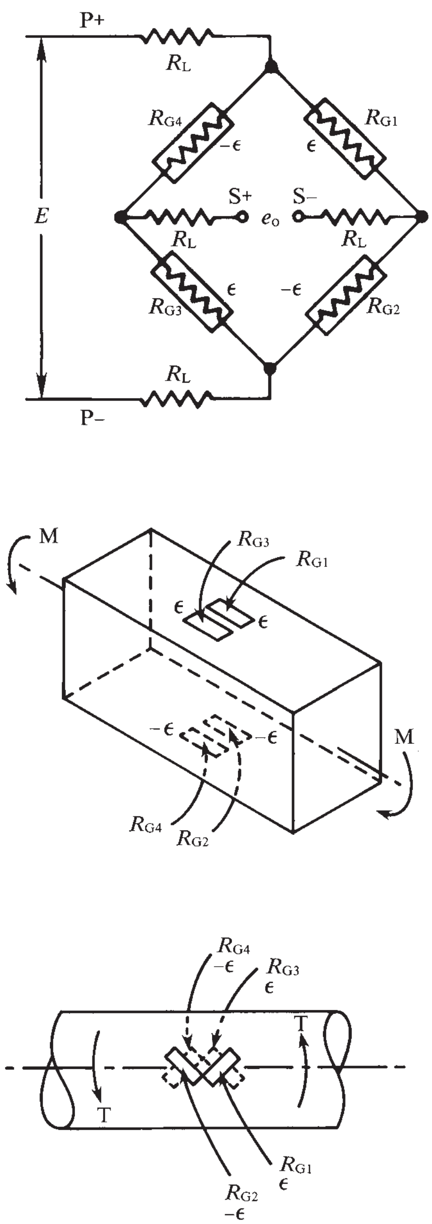

7b – Bending beam, with Poisson gage orientation: N = 2(1 + $\nu$)




7c – Uniform uniaxial stress, with Poisson gage orientation: N = 2(1 + $\nu$)

TECH NOTE

# Shunt Calibration of Strain Gage Instrumentation




7d – Bending or torsion: $N = 4$.

bending and direct stress, respectively, the number of active gages is expressed by $N = 2(1 + \nu)$. This value is substituted into Equation (8) or Equation (9) when calculating the calibration resistor to simulate a surface strain of $\varepsilon_S$. The physical arrangement of the gages is the same in both cases; but, as indicated by the equivalent-circuit diagrams, the gages are positioned differently in the bridge circuit to produce the desired signal in each instance. For applications involving pure bending or torsion, the bridge output signal can be increased further with the gage and circuit configurations illustrated in Figure 7d. Since all four gages are fully active in these examples, $N = 4$ for substitution into Equations (8) or (9).

In general, the shunt-calibration relationships appearing in this section are limited in application to the simulation of low strain magnitudes, since nonlinearity effects in the Wheatstone bridge circuit have been ignored. The equations given here are intended primarily for scaling the output of an instrument to register the same strain magnitude that it would if the selected gage were subjected to an actual strain equal to the simulated strain. This mode of shunt calibration offers a simple, convenient means for eliminating the effects of leadwire desensitization and accounting for more than one active gage ($N > 1$) in the bridge circuit.

For calibration at strain levels higher than about $2000\mu\varepsilon$, or for precise evaluation of instrument accuracy, it is ordinarily necessary to incorporate the effects of Wheatstone bridge nonlinearity in the shunt-calibration relationships. Nonlinearity considerations are treated in Section IV, and application examples given in Section V.

## IV. Wheatstone Bridge Nonlinearity

As described in TN-507, the common practice with modern strain gage instruments is to operate the Wheatstone bridge circuit in a resistively unbalanced mode during strain measurement. In some instruments, the resulting bridge output voltage is read directly as a measure of the strain-induced resistance change(s) in one or more of the bridge arms. In others, the bridge output signal is "unbalanced" (nulled) by injecting an equal and opposite voltage from a separate circuit which is powered by an equal supply voltage.

The cause of the nonlinear behavior (when it occurs) can be demonstrated by reexamining Equation (1a), with reference to Figure 1. The bridge output voltage under any initial condition can be expressed as:

$$\left(\frac{e_O}{E}\right)_1 = \frac{R_1}{R_1 + R_2} - \frac{R_4}{R_4 + R_3} \qquad (10)$$

Considering, for the moment, resistance changes, in $R_1$ and $R_2$ (composing the right-handed branch of the bridge circuit), the output voltage after such changes is:

$$\left(\frac{e_O}{E}\right)_2 = \frac{R_1 + \Delta R_1}{R_1 + R_2 + \Delta R_1 + \Delta R_2} - \frac{R_4}{R_4 + R_3} \qquad (11)$$

# Shunt Calibration of Strain Gage Instrumentation

The change in the output signal from the bridge (or the nulling voltage) is then:

$$\Delta\left(\frac{e_O}{E}\right) = \left(\frac{e_O}{E}\right)_2 - \left(\frac{e_O}{E}\right)_1 = \frac{R_1 + \Delta R_1}{R_1 + R_2 + \Delta R_1 + \Delta R_2} - \frac{R_1}{R_1 + R_2} \qquad (12)$$

In the usual case, however, $R_1 = R_2 = R_G$, the nominal strain gage resistance. After making this substitution, and reducing,

$$\Delta\left(\frac{e_O}{E}\right) = \frac{\frac{\Delta R_1}{R_G} - \frac{\Delta R_2}{R_G}}{4 + 2\frac{\Delta R_1}{R_G} + 2\frac{\Delta R_2}{R_G}} \qquad (13)$$

For the quarter-bridge circuit with only a single active gage ($R_1$), $\Delta R_2 = 0$ and:

$$\Delta\left(\frac{e_O}{E}\right) = \frac{\frac{\Delta R_1}{R_G}}{4 + 2\frac{\Delta R_1}{R_G}} \qquad (14)$$

Or, introducing the relationship from Equation (4),

$$\Delta\left(\frac{e_O}{E}\right) = \frac{F_G\varepsilon}{4 + 2F_G\varepsilon} \qquad (15)$$

It is evident from Equations (14) and (15) that in a quarter-bridge circuit the output is a nonlinear function of the resistance change and the strain — due to the presence of the second term in the denominator. The nonlinearity reflects the fact that as the gage resistance changes, the current through $R_1$ and $R_2$ also changes, in the opposite direction of the resistance change. For typical working strain levels, the quantity $2F_G\varepsilon$ in Equation (15) is very small compared to 4, and the nonlinearity can usually be ignored. When measuring large strains, or when the greatest precision is required, the indicated strain must be corrected for the nonlinearity. With VMM Systems 5000, 6000 and 7000, the correction can be made automatically, and at all strain levels.

Returning to the more general expression for the output of a half-bridge [Equation (13)], it can be seen that the nonlinearity terms in the denominator can be eliminated only by setting $\Delta R_2 = \Delta R_1$. Then, with the resistance changes in $R_1$ and $R_2$ numerically equal, but opposite in sign, Equation (13) reduces to the linear expression:

$$\Delta\left(\frac{e_O}{E}\right) = \frac{2\frac{\Delta R_1}{R_G}}{4} = \frac{\frac{\Delta R_1}{R_G}}{2} \qquad (16)$$

Thus, when the separate resistance changes in $R_1$ and $R_2$ are such that the total series resistance is unchanged, the current through $R_1$ and $R_2$ remains constant, and the bridge output is proportional to the resistance change. A common application of this condition occurs when a beam in bending is instrumented with a strain gage on the convex side and another, mounted directly opposite, on the concave side. The strains sensed by the two gages are then equal in magnitude and opposite in sign ($\varepsilon_2 = -\varepsilon_1$). If the two gages are connected in a half-bridge circuit, as in Figure 5c, the conditions required for Equation (16) are satisfied, and the bridge output, expressed in strain units, is:

$$\Delta\left(\frac{e_O}{E}\right) = \frac{F_G|\varepsilon|}{2}$$ where $|\varepsilon|$ represents the absolute value of strain in either gage. (17)

This demonstration has dealt with only bridge arms $R_1$ and $R_2$ in Figure 1, but it applies equally to arms $R_3$ and $R_4$. The principle can be generalized as follows: any combination of strains and resultant resistance changes in two series bridge arms ($R_1$ and $R_2$, or $R_3$ and $R_4$) which causes the current in that branch of the bridge circuit to change will introduce nonlinearity into the output. Relationships giving the nonlinearity errors for a variety of commonly used circuit arrangements are given in TN-507.

With the foregoing principle in mind, we are now in a position to consider the effect of Wheatstone bridge nonlinearity on shunt calibration. It should first be noted that, in normal practice, only one arm of the bridge is shunted at a time; and it is never possible, by shunting, to produce equal and opposite resistance changes in $R_1$ and $R_2$, or in $R_3$ and $R_4$. Thus, the shunt-calibration procedure always results in nonlinear, quarter-bridge operation — regardless of how the bridge circuit functions during actual strain measurement. In Figure 5c, for instance, the bridge output during strain measurement is proportional to the surface strain, as indicated by Equation (17). When either gage is shunted by a calibration resistor, however, the output is nonlinearly related to the simulated strain according to Equation (15). As a result, shunting, say, $R_{G1}$ in Figure 5c does not exactly simulate, in terms of bridge output voltage, the behavior of the gage during strain measurement.

As noted earlier, the effect of the nonlinearity is small when the strains (actual or simulated) are small. For such cases, the relationships given in Section III are adequate to permit calculating the shunt resistor size to simulate a

# Shunt Calibration of Strain Gage Instrumentation

given strain magnitude. Because of this, it is preferable to perform instrument scaling at modest strain levels where the nonlinearity error is negligible.

When instrument scaling is done at higher strain levels, it is generally necessary to use special relationships, given in Section V, to precisely simulate gage behavior by shunt calibration. There is one notable exception to the latter statement, however. The bridge output due to shunting a single gage is indistinguishable from that of a quarter-bridge circuit with the gage strained in compression. For this special (but common) case, the simulation is exact at all compressive strain levels because the nonlinearity due to shunting is the same as that caused by compressive strain in the gage. As shown in the Appendix, Equations (5) to (7) are thus appropriate for compressive scaling of quarter-bridge circuits at any level of strain.[4] The same is true, of course, for external half- and full-bridge circuits where there is only a single active gage, with the remaining bridge arms used for compensation and/or bridge-completion purposes.

Both Sections III and V are limited in scope to the subject of instrument scaling by gage simulation. Shunt calibration for instrument *verification* is treated separately in Section VI. For scaling applications, the size of the shunt-calibration resistor is selected so that the bridge output voltage is the same for both simulated and actual strains of the same magnitude. Therefore, when the Wheatstone bridge arrangement is intrinsically nonlinear (as in Figure 5b, for instance), and the strain level is high, the instrument indication is accurate only at the simulated strain level. Subsequent correction may be needed if the instrument is to accurately register smaller or larger strains.

## V. Instrument Scaling for Large Strains

It was demonstrated in the preceding section that Wheatstone bridge nonlinearity must generally be considered when shunt calibration is used for instrument scaling at high strain levels. Under such conditions, errors in gage simulation arise whenever the nonlinearity which is inherent in shunt calibration differs from that during actual strain measurement. The relationships given in this section for quarter-, half-, and full-bridge circuits provide for precisely simulating the strain gage output at any level of strain, low or high.[5] Thus, the instrument scaling will also be precise when the gain or gage factor control is adjusted to register the simulated strain. It must be kept in mind, however, that if the strain-measuring circuit arrangement is nonlinear (as in Figures 4, 5b and 7c), precise scaling is achieved only at the simulated strain level. At any other level of strain, some degree of error will be present due to the nonlinearity.

### Quarter-Bridge Circuit

The quarter-bridge circuit, with a single active gage, is widely used in experimental stress analysis. When instrument scaling is done by connecting a shunt-calibration resistor directly across the gage, the simulation of compressive strain is exact at all strain levels. This is true because the nonlinearity in shunt calibration is the same as that during strain measurement. For such cases, the proper shunt-calibration resistor to simulate a given strain magnitude can be obtained directly from Table 1, or calculated from Equation (7) of Section II. After instrument scaling, the indicated strain will be correct at the magnitude of the calibration strain, but slightly in error at other strain levels because of the nonlinearity. For most practical applications, the corrected strain at any different strain level can be calculated from:

$$\varepsilon = \frac{2\tilde{\varepsilon}}{2 + F_G\left(\varepsilon_S - \tilde{\varepsilon}\right)} \tag{18}$$

where: $\varepsilon$ = corrected strain
$\varepsilon_S$ = calibration strain
$\tilde{\varepsilon}$ = indicated strain
$F_G$ = gage factor of strain gage

Since the effect of leadwire resistance on bridge circuit nonlinearity is normally very small, terms involving $R_L$ have been omitted from Equation (18). If the leadwire resistance is a significant fraction of the gage resistance, however, Equation (18) tends to overcorrect for the nonlinearity. In such cases, the following complete relationship can be used to obtain more accurate correction:

$$\varepsilon = \frac{2\left(1 + \frac{R_L}{R_G}\right)\tilde{\varepsilon}}{2\left(1 + \frac{R_L}{R_G}\right) + F_G\left(\varepsilon_S - \tilde{\varepsilon}\right)} \tag{18a}$$

Shunting the dummy arm of the bridge (see Figure 4) produces an upscale signal, and can be used to simulate a tensile strain in the active gage. For the simulation to be exact, however, a special shunt-calibration relationship is required, because the nonlinearity in tension is different from that in compression. If the active gage were subjected to an actual tensile strain, the resistance of the right-hand branch of the bridge in Figure 4 would rise, and the current

[4] Since the nonlinearity due to a resistance increase is different than for a decrease, precise simulation of a high tensile strain requires a special relationship, as demonstrated in Section V.

[5] The subject relationships are "precise" or "exact" with respect to the specified parameters such as $R_G$, $R_C$, and $F_G$. The effects of tolerances on these quantities are discussed in Section VII, "Accuracy Considerations".

# Shunt Calibration of Strain Gage Instrumentation

would decrease correspondingly. However, when the dummy arm of the bridge is shunted, the resistance of the branch decreases, and the current rises. This difference can be accounted for by calculating the calibration resistor so that the bridge output voltage due to shunting the dummy is the same as that for a preselected tensile calibration strain in the active gage. The procedure for doing so is demonstrated by the following derivation where, for the sake of simplicity, the effect of leadwire resistance is temporarily ignored ($RL = 0$).

Equation (14) in Section IV gives the output voltage from a resistance change in the active gage ($R_1$). The negative of the same relationship applies to a change in the dummy arm, $R_2$. Thus,

$$\left(\frac{e_O}{E}\right)_1 = \frac{\frac{\Delta R_1}{R_G}}{4 + 2\frac{\Delta R_1}{R_G}} \tag{19}$$

$$\left(\frac{e_O}{E}\right)_2 = \frac{-\frac{\Delta R_2}{R_G}}{4 + 2\frac{\Delta R_2}{R_G}} \tag{20}$$

The unit resistance change in the active gage due to a simulated tensile strain $\varepsilon_{ST1}$ is:

$$\frac{\Delta R_1}{R_G} = F_G\ \varepsilon_{ST1}$$

Therefore,

$$\left(\frac{e_O}{E}\right)_1 = \frac{F_G\ \varepsilon_{ST1}}{4 + 2F_G\ \varepsilon_{ST1}} \tag{21}$$

On the other hand, the resistance change in the dummy, $R_2$, is produced by shunting with a calibration resistor, $R_C$. From Equation (3),

$$\frac{\Delta R_2}{R_G} = -\frac{R_G}{R_G + R_C}$$

Substituting into Equation (20),

$$\left(\frac{e_O}{E}\right)_2 = \frac{\frac{R_G}{R_G + R_C}}{4 - 2\frac{R_G}{R_G + R_C}} = \frac{R_G}{2R_G + 4R_C} \tag{22}$$

Equating the two expressions for output voltage,

$$\frac{F_G\ \varepsilon_{ST1}}{4 + 2F_G\ \varepsilon_{ST1}} = \frac{R_G}{2R_G + 4R_C}$$

And, solving for $R_C$,

$$R_C = \frac{R_G}{F_G\ \varepsilon_{ST1}} = \frac{R_G \text{ x } 10^6}{F_G\ \varepsilon_{ST1}(\mu)} \tag{23}$$

For the majority of routine applications, any desired tensile strain in the active gage can be simulated quite accurately by shunting the dummy gage with the calibration resistor specified by Equation (23). This relationship can be compared to Equation (7) to see the difference between simulating tensile and compressive strains in the active gage. After scaling for a given simulated tensile strain, the instrument indication will be correspondingly accurate for only that tensile strain magnitude. Measurements at other strain levels (tension or compression) can be corrected, if necessary, with Equation (18) or (18a).

In extreme cases, when the simulated strain is very large, and the leadwire resistance is not a negligible fraction of the gage resistance, slightly greater accuracy in tensile strain simulation can be achieved by incorporating the leadwire resistance in the derivation of Equation (23). The complete expression for the calibration resistor becomes:

$$R_C = \frac{R_G}{F_G\ \varepsilon_{ST1}} - R_G\left(\frac{\frac{R_L}{R_G}}{1 + \frac{R_L}{R_G}}\right) \tag{23a}$$

The second term in Equation (23a) can never be greater in magnitude than $R_G$; and, for typical strain levels, is negligible compared to the first term — irrespective of the leadwire resistance. Thus Equation (23) is normally the appropriate relationship for the shunt resistor used in upscale calibration. The small error associated with

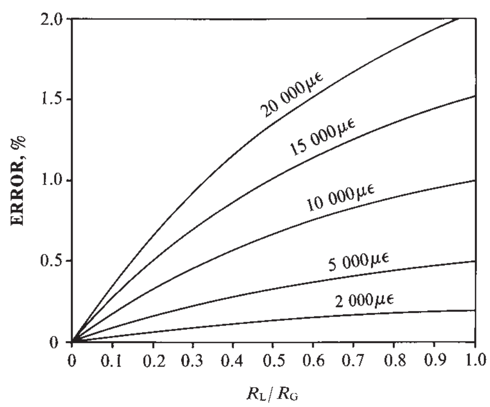



Figure 8 – Percent error from using Equation (23) instead of Equation (23a). $F_G = 2.0$.

TECH NOTE

# Shunt Calibration of Strain Gage Instrumentation

Equation (23) is plotted in Figure 8 as a guide to the very rare circumstances when Equation (23a) might be necessary.

It is worth noting, for quarter-bridge circuits, that scaling the instrument by shunting the internal dummy gage (which is usually a stable precision resistor) can offer distinct advantages in calibration accuracy. It is common practice, for instance, to calculate or select the value of the shunt-calibration resistor on the basis of the nominal gage resistance. But the resistance of the installed gage generally differs from the nominal, due both to its initial resistance tolerance and to a further change in resistance during installation. When this occurs, and the active gage is shunted for compression scaling, the simulated magnitude is in error accordingly. The extent of the error can be approximated by the method given in Section VII, "Accuracy Considerations".

One technique for avoiding most of the error due to deviation in the gage resistance is to temporarily replace the active gage in the bridge circuit with a precision resistor equal to the nominal resistance of the gage. The instrument is then scaled (in compression) by shunting the fixed resistor with a calibration resistor calculated from Equation (7). After scaling, the active gage is reconnected to the bridge circuit. It is usually much more convenient, however, and about equally accurate, to scale in the tension direction by simply shunting the internal dummy with a resistor calculated from Equation (23). When the leadwire resistance is negligible, this procedure is exact, and independent of the installed gage resistance. Even with modest leadwire resistance (say, less than $R_G$ /10), the error due to a few ohms of gage resistance deviation is small enough to be ignored. In case of doubt, the installed gage resistance should be measured. If the resistance is significantly beyond the manufacturer's tolerance, one of the two foregoing procedures should always be used for shunt calibration.

## Half-Bridge Circuits

When measuring the maximum principal strain in a known uniaxial stress state, a simple means for assuring effective temperature compensation is to mount a second gage adjacent and perpendicular to the primary gage, and connect the two gages in a half-bridge circuit as shown in Figure 5b. Such an arrangement is said to have $N = 1 + \nu$ active gages, since the bridge output is increased by that factor. The circuit behavior is nonlinear, however, because the resistance changes in the two active gages are not equal and opposite.

It is assumed in the following that $R_{G1}$ in Figure 5b represents the primary gage, and that the object is to scale the instrument to register the test-surface strain under that gage. Whether shunting $R_{G1}$ to simulate compression in the primary gage, or shunting $R_{G2}$ to simulate tension, the nonlinearity during scaling is different from that during actual strain measurement. Thus, two different shunt-calibration relationships are required for precise strain simulation, as in the case of the quarter-bridge circuit. These relationships are developed in the same manner as before; that is, by enforcing the condition that the bridge output voltage be identical, whether scaling to a simulated strain level or measuring the same surface strain with the primary gage.

To simulate a compressive surface strain, $\varepsilon_{SC1}$, by shunting $R_{G1}$ in Figure 5b, the calibration resistor is calculated from:

$$R_C = \frac{R_G}{F_G\ \varepsilon_{SC1}(1+\nu)} - R_G\left[1 - \frac{\nu}{(1+\nu)\left(1+\frac{R_L}{R_G}\right)}\right] \qquad (24)$$

And, for a simulated tensile strain, $\varepsilon_{ST1}$, generated by shunting $R_{G2}$.

$$R_C = \frac{R_G}{F_G\ \varepsilon_{ST1}(1+\nu)} - R_G\left[1 - \frac{1}{(1+\nu)\left(1+\frac{R_L}{R_G}\right)}\right] \qquad (25)$$

When substituting into Equations (24) and (25), the signs of the simulated strains must always be carried, of course. Also, it is apparent that leadwire resistance, if present, affects the nonlinear behavior, and must be known to permit exact simulation. Since the expressions are relatively insensitive to the quantity $R_L/R_G$, precision measurement of the leadwire resistance is not ordinarily required. After scaling the instrument at a simulated strain, $\varepsilon_S$, the registered strain is precisely correct for only that same magnitude. When necessary, the corrected strain at any level can be calculated from:

$$\varepsilon = \frac{2\left(1+\frac{R_L}{R_G}\right)\tilde{\varepsilon}}{2\left(1+\frac{R_L}{R_G}\right) + F_G(1-\nu)(\varepsilon_S - \tilde{\varepsilon})} \qquad (26)$$

The notation in Equation (26) is the same as in Equation (18) for the quarter-bridge circuit.

Another half-bridge application of interest is illustrated in Figure 5c, where directly opposed strain gages are installed on the convex and concave sides of a rectangular-cross-section beam in bending. In this instance, the strains in $R_{G1}$ and $R_{G2}$ are always equal and opposite, if only bending

TECH NOTE

# Shunt Calibration of Strain Gage Instrumentation

occurs. As a result, the bridge behavior during strain measurement is linear; and, after scaling at a particular strain level, remains equally precise at all other strain magnitudes. Instrument scaling by shunt calibration is a nonlinear procedure, however, because there is a resistance change in only a single bridge arm. The simplest approach is to perform the scaling at a modest strain level where the error due to calibration nonlinearity is negligible. After scaling with Equation (8), the instrument will register any other strain with the same relative precision.

If scaling at a high strain level is necessary, the calibration resistor can be calculated as follows to provide exact simulation of a surface strain, $\varepsilon_S$ in $R_{G1}$ (when accompanied by the strain $-\varepsilon_S$ in $R_{G2}$):

$$R_C = \frac{R_G}{sF_G|\varepsilon_S|} - R_G\left[1 - \frac{1}{2\left(1+\frac{R_L}{R_G}\right)}\right] \tag{27}$$

where: $|\varepsilon_S|$ = absolute value of calibration strain.

Equation (27) is suitable for either upscale or downscale shunt calibration. If the leadwire resistance is negligible, the relationship reduces to:

$$R_C = \frac{R_G}{sF_G|\varepsilon_S|} - \frac{R_G}{2} \tag{28}$$

Because the bridge output voltage varies linearly with strain when actual strains are being measured, no further correction is required.

## Full-Bridge Circuits

When feasible, use of the full-bridge circuit offers several advantages, including a better signal-to-noise ratio. Typical applications are: beams in bending, shafts in torsion, and axially loaded columns and tension links. Although the simple examples described here do not incorporate the circuit refinements characteristic of commercial transducers, it is common practice to infer the magnitudes of mechanical variables such as bending moment, torque, and force from the full-bridge strain measurement.

Three representative full-bridge arrangements, illustrated in Figures 7b, 7c, and 7d, are treated here. The circuits in Figures 7b and 7d (for bending and torsion) have essentially the same characteristics, and can be grouped together for shunt-calibration purposes. In each of these circuits, the bridge output voltage varies linearly with strain, since equal and opposite resistance changes occur in arms 1 and 2, and in arms 3 and 4. The nonlinearity of shunt calibration must be accounted for, however, to achieve exact strain stimulation at large strains. The proper calibration resistor to simulate a given surface strain (e.g., the longitudinal strain, in the case of the beam) can be calculated from the following:

$$R_C = \frac{R_G}{NF_G|\varepsilon_S|} - R_G\left[\frac{1+3\frac{R_L}{R_G}}{2\left(1+2\frac{R_L}{R_G}\right)}\right] \tag{29}$$

Once calibrated according to Equation (29), an accurate instrument will register the correct strain at any other strain magnitude. As in the case of the half-bridge circuit, the leadwire resistance is present in the calibration relationship, but does not need to be known with high precision.

The arrangement shown in Figure 7c, for a centrally loaded column or tension member, is somewhat more complex. It can be seen from the figure that the bridge currents change with applied strain, and thus the output voltage is a nonlinear function of strain even before calibrating with a shunt resistor. Because the nonlinearity is different in tension and compression, separate calibration equations are required as follows:

To simulate compression in $R_{G1}$,

$$R_C = \frac{-R_G}{2(1+\nu)F_G\varepsilon_{SC1}} - R_G\left[\frac{3(1+\nu)\left(1+2\frac{R_L}{R_G}\right)-2\nu}{4(1+\nu)\left(1+2\frac{R_L}{R_G}\right)}\right] \tag{30}$$

And, for tension in $R_{G1}$,

$$R_C = \frac{R_G}{2(1+\nu)F_G\varepsilon_{ST1}} - R_G\left[\frac{3(1+\nu)\left(1+2\frac{R_L}{R_G}\right)-2}{4(1+\nu)\left(1+2\frac{R_L}{R_G}\right)}\right] \tag{31}$$

Equations (30) and (31) provide for correctly simulating the longitudinal surface strain in compression and tension, respectively, at any strain magnitude. In common with the corresponding half bridge (Figure 5b), the instrument indication at the calibration strain level is precise, but operation at a different strain level will introduce a small error due to bridge-circuit nonlinearity. When necessary, the corrected strain can be calculated from [see Equation (18) for notation]:

# Shunt Calibration of Strain Gage Instrumentation

$$\varepsilon = \frac{2\left(1+2\dfrac{R_L}{R_G}\right)\tilde{\varepsilon}}{2\left(1+2\dfrac{R_L}{R_G}\right)+F_G(1-\nu)(\varepsilon_S-\tilde{\varepsilon})} \qquad (32)$$

## VI. Instrument Verification

The shunt-calibration procedures described in Sections III and V are intended specifically for instrument scaling purposes; that is, for adjusting the instrument output to match a given simulated surface strain. They are not directly related to the question of verifying the linearity and/or absolute accuracy of a strain-measuring instrument. It is implicitly assumed in the preceding sections of this each Tech Note that the instrument involved is perfectly linear in its response characteristics and, if direct-indicating, is perfectly accurate. In practice, however, it is necessary to periodically verify the accuracy of the instrument by calibration; and methods for accomplishing this are given here.[6]

By far the most reliably accurate means for instrument verification is through the use of a laboratory-standard calibrator such as our Model 1550A. This instrument, which incorporates true tension and compression strain simulation, provides precision calibration of strain indicators to an accuracy of 0.025 percent. It also eliminates errors due to the tolerances on the strain gage and shunt resistances. The calibrator is equipped with three decades of switches, which permit rapid calibration in small steps over a very wide strain range (to ~100000$\mu\varepsilon$).

Whether verification of the strain indicator is to be done with a precision calibrator or by shunt calibration, it is important that the procedure be unaffected by leadwire resistance. When verifying instrument accuracy with the Model 1550A, for instance, the calibrator should be connected to the strain indicator with short leads of generous wire size. Similarly, with shunt calibration, the leadwire resistance in the shunted bridge arm should be negligibly small. This can be accomplished, for calibration purposes, by connecting an installed strain gage or a stable precision resistor directly across the active gage terminals of the strain indicator. Either the active or dummy arm of the bridge circuit can then be shunted to produce, correspondingly, a downscale or upscale calibration signal. If the active arm is a strain gage, and is to be shunted, the installed resistance of the gage must be known accurately.

An alternative approach, which eliminates the effect of leadwire resistance, is to shunt one arm of the internal half-bridge commonly found in conventional strain gage instruments. This procedure requires, of course, that the resistances of the internal bridge arms be known. In addition, it requires that the internal half-bridge be isolated from any balance circuitry which may be present, or that the effects of such circuitry be incorporated in the shunt-calibration calculations. In any case, the instruction manual and circuit diagram for the instrument should be consulted before attempting to calibrate by shunting the internal half-bridge.

The calibration relationship for instrument verification is based on different reasoning than it is for instrument scaling. In scaling applications (Sections III and V), the calibration resistor is calculated to develop the same bridge output voltage that would occur when a strain gage of specified gage factor is subjected to a given strain. The instrument gage factor or gain control is then adjusted to register the simulated strain. The effects of signal loss due to leadwire resistance, or signal increase form multiple active gages, are thus compensated for. With this technique, the final setting of the gage-factor or gain control is determined only by the external circuit parameters; and, in the case of a strain indicator, for example, the resulting gage factor setting of the instrument would normally be quite different from that of the strain gage.

In contrast, when calibrating for instrument verification purposes, the instrument gage factor or gain is ordinarily preset to some convenient value. The verification relationship is then written to express the registered strain (in a perfectly accurate instrument) as a function of the shunt resistor used to synthesize the strain signal. It will be seen that the gage factor of the strain gage itself is not involved in this process. Nor are other external circuit parameters, except the initial resistance of the shunted bridge arm, which is usually the nominal resistance of a strain gage.

In an ideal instrument, the registered strain is related to the bridge output voltage by:

$$\varepsilon_I = C \text{ x } \frac{e_O}{E} \qquad (33)$$

where: $\varepsilon_I$ = strain magnitude indicated or registered by the ideal instrument.

$C$ = instrument constant — at a fixed setting of the gain or gage-factor control.

But the bridge output caused by a unit resistance change in one arm can be expressed as:

$$\frac{e_O}{E} = \pm\frac{\dfrac{\Delta R}{R_G}}{4+2\dfrac{\Delta R}{R_G}} \qquad (34)$$

In Equation (34), the choice of sign depends on which arm is being shunted. Referring to Figure 1, the sign is positive for $R_1$ or $R_3$, and negative for $R_2$ or $R_4$. The unit resistance

[6] As used in this section only, the term "calibration" thus refers exclusively to the process of instrument verification for linearity or accuracy.

# Shunt Calibration of Strain Gage Instrumentation

change in shunt calibration is always negative, however, and is calculated from Equation (3):

$$\frac{\Delta R}{R_G} = \frac{-R_G}{R_G + R_C}$$

Substituting into Equation (34), and simplifying,

$$\frac{e_O}{E} = \mp \frac{R_G}{4R_C + 2R_G} \tag{35}$$

Thus, the bridge output is negative when shunting $R_1$ or $R_3$, and positive for $R_2$ or $R_4$.

Substituting Equation (35) into Equation (33),

$$\varepsilon_I = \mp C \text{ x } \frac{R_G}{4R_C + 2R_G}$$

Or, in general, when shunting any arm of initial resistance $R_G$,

$$|\varepsilon_I| = C \text{ x } \frac{R_G}{4R_C + 2R_G} \tag{36}$$

And,

$$R_C = \frac{C}{4} \text{x} \frac{R_G}{|\varepsilon_I|} - \frac{R_G}{2} \tag{37}$$

where: $|\varepsilon_I|$ = absolute value of registered strain (ideal).

In the case of a strain indicator with a gage-factor control, the instrument is designed so that $C = 4/F_I$, where $F_I$ is the instrument gage factor setting. Then,

$$R_C = \frac{R_G}{F_I \text{ x } |\varepsilon_I|} - \frac{R_G}{2} \tag{38}$$

When one arm of the bridge is shunted by a calibration resistor calculated from Equation (38), the instrument should indicate the synthesized strain, $\varepsilon_I$. Failure to do so by more than the tolerances on $R_G$ and $R_C$ is indicative of instrument error.

Instead of a strain indicator, the instrumentation may consist of a signal-conditioning amplifier. This type of instrument is normally equipped with a gain control rather than a gage-factor control. Its output is simply a voltage which can then be supplied to an oscillograph or other device for recording. In the ideal instrument, the voltage at any gain setting should be strictly proportional to the bridge output signal. Thus, corresponding to Equation (33),

$$V = C\frac{e_O}{E} \tag{39}$$

The object of calibration in this instance is to verify the instrument linearity; that is, to test whether $C$ is, in fact, constant. Calibration is accomplished by comparing the measured instrument output voltage to the bridge output signal at a series of different signal levels. Substituting Equation (35) into Equation (39),

$$V = \mp C \text{ x } \frac{R_G}{4R_C + 2R_G}$$

Or, in general,

$$\frac{|V|}{\left(\dfrac{R_G}{4R_C + 2R_G}\right)} = C \tag{40}$$

After shunting one arm of the bridge with a calibration resistor, $R_C$, the instrument output voltage is measured, and the constant, $C$, calculated from Equation (39). This operation is repeated at two or more different signal levels by successively shunting with appropriate calibration resistors. If the instrument is linear, variations in the calculated value of the instrument constant should not be greater than the tolerances on the parameters in Equation (40).

## VII. Accuracy Consideration

As described in the preceding sections, shunt calibration can be used for either system scaling or instrument verification purposes. In both cases, the greatest attainable accuracy with the procedure is limited by errors (deviations from the nominal or assumed value) in the variables which enter into the calibration calculations. The error sensitivity of the method can be demonstrated most easily with a generalized form of the basic shunt-calibration relationship [see Equations (3), (4) and (5)].

Let $R_1$ in Equation (3) represent the *actual* resistance of the strain gage, after installation. The factor $R_G$ in Equation (4) is replaced by a numerical constant, $C$, to emphasize the fact that the *nominal* resistance of the gage is not changed by gage installation. Then, the relationships in Equations (3) and (4) can be reexpressed as:

$$\Delta R = \frac{R_1^2}{R_1 + R_C} \tag{3a}$$

$$\Delta R = C F_G \varepsilon \tag{4a}$$

Combining Equations (3a) and (4a), and solving for the simulated strain,

TECH NOTE

## Shunt Calibration of Strain Gage Instrumentation

$$\left|\varepsilon_S\right| = \frac{R_1^2}{CF_G\left(R_1 + R_C\right)} \quad (41)$$

The total differential of the simulated strain can be written:

$$d\varepsilon_S = \frac{\partial\varepsilon_S}{\partial R_1}\partial R_1 + \frac{\partial\varepsilon_S}{\partial R_C}\partial R_C + \frac{\partial\varepsilon_S}{\partial F_G}\partial F_G$$

After performing the partial differentiations and dividing through by $\varepsilon_S = R_1^2/CF_G\,(R_1 + R_C)$,

$$\frac{d\varepsilon_S}{\varepsilon_S} = \left(\frac{R_1 + 2R_C}{R_1 + R_C}\right)\frac{dR_1}{R_1} - \left(\frac{R_C}{R_1 + R_C}\right)\frac{dR_C}{R_C} - \frac{dF_G}{F_G} \quad (42)$$

For small deviations, the differentials can be replaced by finite differences, or:

$$\frac{\Delta\varepsilon_S}{\varepsilon_S} = \left(\frac{R_1 + 2R_C}{R_1 + R_C}\right)\frac{\Delta R_1}{R_1} - \left(\frac{R_C}{R_1 + R_C}\right)\frac{\Delta R_C}{R_C} - \frac{\Delta F_G}{F_G} \quad (43)$$

When multiplied by 100, Equation (43) gives the percent error in the simulated strain as a function of the errors or deviations in $R_1$ (the actual gage resistance), $R_C$, and $F_G$. Since $R_C$ is ordinarily very large compared to $R_1$, it can be seen that the percent error in simulated strain is about twice that in the gage resistance, and is approximately equal to that in the calibration resistor and gage factor (although the signs may differ). In practice, the errors in $R_1$, $R_C$, and $F_G$ vary independently over their respective ranges of tolerance or uncertainty. Thus, they may tend to be self-canceling on some occasions; and, at other times, may be additive. The worst-case errors in simulated strain occur when $\Delta R_1$ is positive while $\Delta R_C$ and $\Delta F_G$ are negative, and vice versa. These conditions can be combined into a single expression by employing the absolute values of the errors:

$$\left|\frac{\Delta\varepsilon_S}{\varepsilon_S}_{\text{MAX}}\right| =$$

$$\left(\frac{R_1 + 2R_C}{R_1 + R_C}\right)\left|\frac{\Delta R_1}{R_1}\right| + \left(\frac{R_C}{R_1 + R_C}\right)\left|\frac{\Delta R_C}{R_C}\right| + \left|\frac{\Delta F_G}{F_G}\right| \quad (44)$$

Equation (44) permits calculating the extreme error in simulated strain from the extreme errors in the other variables. Practically, however, the extreme errors in $R_G$, $R_C$, and $F_G$ would occur only rarely at the same time, and with the required combination of signs, to be fully additive. A better measure of the approximate uncertainty (expected error range) in $\varepsilon_S$ as a function of the uncertainties or tolerances on the other three quantities can be obtained by an adaptation from the theory of error propagation. The latter theory is not strictly applicable in this case because the individual error distributions are unknown, are probably different from one another, and may otherwise violate statistical requirements of the method. However, if the uncertainties in each variable represent about the same number of standard deviations, the following expression should give a more realistic estimate of the uncertainty in $\varepsilon_S$ than Equation (44):

$$\frac{U\varepsilon_S}{\varepsilon_S} = \quad (45)$$

$$\sqrt{\left(\frac{R_1 + 2R_C}{R_1 + R_C}\right)^2\left(\frac{UR_1}{R_1}\right)^2 + \left(\frac{R_C}{R_1 + R_C}\right)\left(\frac{UR_C}{R_C}\right)^2 + \left(\frac{UF_G}{F_G}\right)^2}$$

where: $\dfrac{U_X}{X}$ = percent uncertainty in variable X.

As a numerical example, assume that a 350-ohm gage with a gage factor of 2.0 is to be shunted to simulate a strain of $2000\mu\varepsilon$. From Equation (7).

$$R_C = \frac{350 \text{ x } 10^6}{2.0 \text{ x } 2000} - 350 = 87150\,\text{ohms}$$

This calibration resistor can be found in Table 1; and, if supplied by Vishay Micro-Measurements, has a resistance tolerance of ±0.01%. Assume also that the selected gage type has tolerances on its gage factor and resistance of ±1% and ±0.3%, respectively. Since the gage resistance may have been shifted during installation, however, the uncertainty in the installed resistance should normally be taken somewhat larger — say, to be conservative, ±1%. Substituting into Equation (45),

$$\frac{U\varepsilon_S}{\varepsilon_S} = \sqrt{(1.996)^2 \text{ x } (1.0)^2 + (0.996)^2 \text{ x } (0.01)^2 + (1.0)^2} = \pm 2.23\%$$

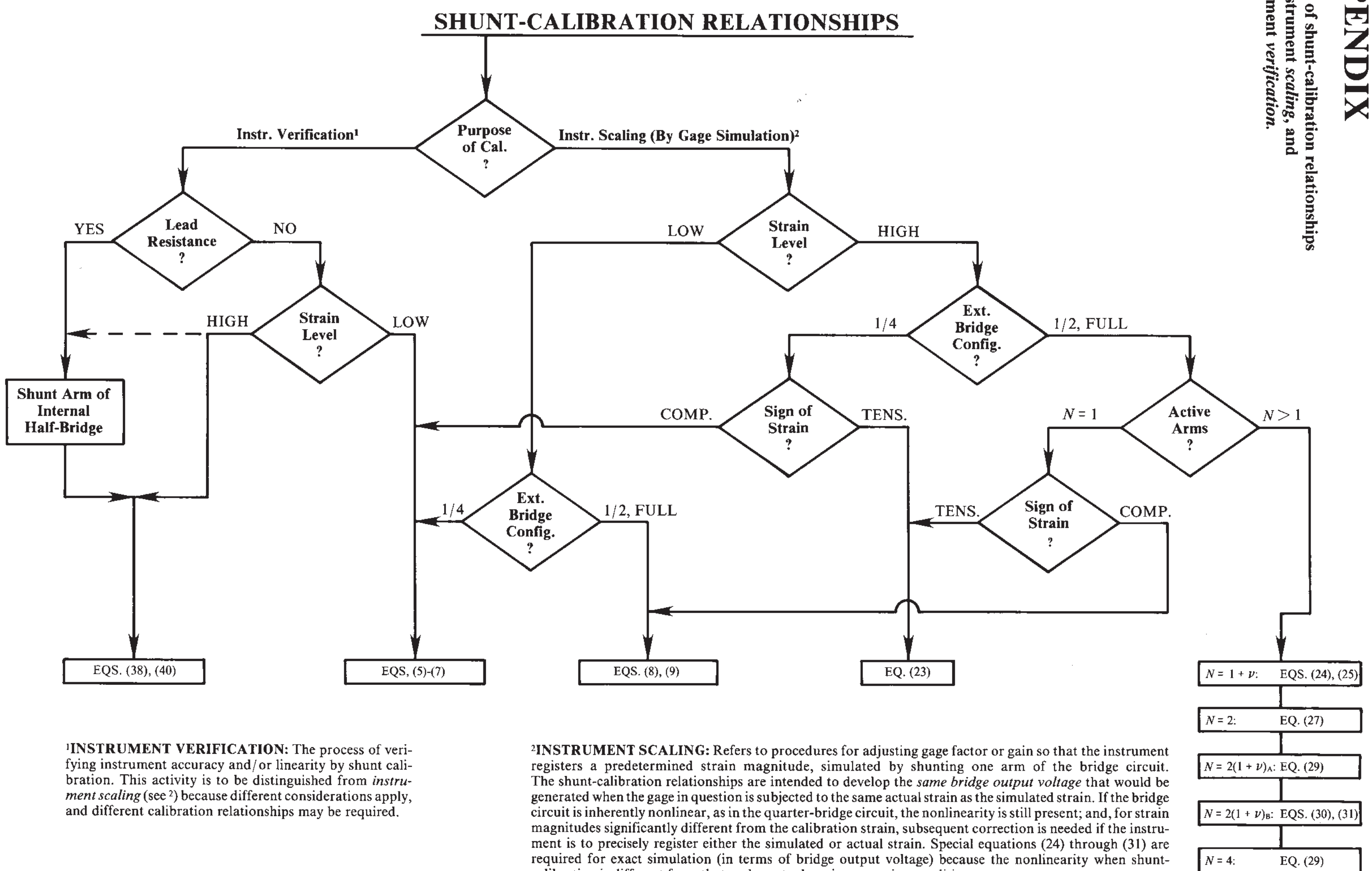
Equations (44) and (45) will be found helpful guides in estimating the precision of shunt calibration. They can also serve in judging whether use of the large-strain relationships in Section V is warranted under any given set of circumstances. Thus, if the intrinsic uncertainty in shunt calibration is many times greater than the refinement obtained by considering large-strain effects, the simpler relationships in Section III may as well be employed.

When necessary, the overall uncertainty can be reduced somewhat by accurately measuring the installed gage resistance and employing this value in the shunt-calibration equations. Or, alternatively, the effect of resistance deviation in the gage can be largely eliminated by the methods described in Section V for quarter-bridge calibration.

# Shunt Calibration of Strain Gage Instrumentation

## APPENDIX

**Selection of shunt-calibration relationships for (a) instrument *scaling*, and (b) instrument *verification*.**




[1]**INSTRUMENT VERIFICATION:** The process of verifying instrument accuracy and/or linearity by shunt calibration. This activity is to be distinguished from *instrument scaling* (see [2]) because different considerations apply, and different calibration relationships may be required.

[2]**INSTRUMENT SCALING:** Refers to procedures for adjusting gage factor or gain so that the instrument registers a predetermined strain magnitude, simulated by shunting one arm of the bridge circuit. The shunt-calibration relationships are intended to develop the *same bridge output voltage* that would be generated when the gage in question is subjected to the same actual strain as the simulated strain. If the bridge circuit is inherently nonlinear, as in the quarter-bridge circuit, the nonlinearity is still present; and, for strain magnitudes significantly different from the calibration strain, subsequent correction is needed if the instrument is to precisely register either the simulated or actual strain. Special equations (24) through (31) are required for exact simulation (in terms of bridge output voltage) because the nonlinearity when shunt-calibrating is different from that under actual strain-measuring conditions.